MAT 534 FALL 2015 REVIEW FOR MIDTERM II

General

The exam will be in class on Thursday, November 12. It will consist of 5 problems and will be a closed book exam on group theory, covering section 1)-10) below.

Material covered in class

1) Rings, divisors of zero, commutative rings, rings with 1, integral domains; finite integral domain is a field.
2) Ideals of a ring R (left, right and two-sided), ring homomorphisms, quotient rings, the isomorphism theorems.
3) Ideals generated by a set $A \subset R$, finitely generated ideals, principal ideals, maximal ideals. In a ring R with 1 every proper ideal is contained in a maximal ideal.
4) R a commutative ring. Definition and characterization of prime and maximal ideals. Every maximal ideal in a commutative ring is a prime ideal. Properties of ideals in P.I.D.
5) R commutative ring, $\emptyset \neq D \subseteq R \setminus \{0\}$ which is closed under multiplication and does not contain zero divisors. Ring $D^{-1}R$ of fractions of D with respect to R; quotient field of R if R is an integral domain and $D = R \setminus \{0\}$.
6) Euclidean domains, P.I.D. and U.F.D., properties and examples. Definition of a prime and irreducible elements in integral domain. Every prime is irreducible and in P.I.D. every irreducible is maximal (and hence prime). In U.F.D. element is prime iff it is irreducible. Arithmetic of $\mathbb{Z}[\sqrt{-1}]$, the Chinese Remainder Theorem.
7) Polynomial ring $R[x]$, R integral domain, relation between prime and maximal ideals in R and $R[x]$. Polynomial ring $F[x]$ over a field, division of polynomials, roots, irreducible polynomials. Every finite subgroup of a multiplicative group of a field is cyclic. $F[x]$ is Euclidean domain. Polynomial rings $R[x]$ over U.F.D., content of a polynomial in $F[x]$, where F is a field of fractions of R, Gauss lemma. $R[x]$ is U.F.D. if R is U.F.D. and primes in $R[x]$ are primes in R and irreducible in $F[x]$ with content 1. Irreducibility criteria.

8) Noetherian rings and Hilbert's Basis Theorem.
9) Modules, submodules, examples. The isomorphism theorems. Direct sums, free modules, modules generated by sets, finitely generated modules, cyclic modules. Modules over P.I.D.
10) Extension of scalars and tensor product of modules.